1948

Determination of minimum temperature from nocturnal radiation

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Annapolis, Maryland: Naval Postgraduate School

http://hdl.handle.net/10945/31607

DETERMINATION OF MINIMUM TEMPERATURE FROM NOCTURNAL RADIATION

by

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Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE IN AEROLOGY

United States Naval Postgraduate School
Annapolis, Maryland
1948

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE IN AEROLOGY

from the
United States Naval Postgraduate School

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PREFACE

The object of this work is to develop a convenient method of forecasting minimum temperature.

This paper was prepared at the United States Postgraduate School for partial fulfillment of the requirements for the degree of master of science in aerology.

The author expresses his thanks to Professor Frank L. Martin, who has rendered valuable aid in advising and editing this paper.

Annapolis, Md. May 15, 1948

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TABLE OF SYMBOLS

A	Heat flux density due to eddy conduction in the air
B	Heat flux density in the ground
C	Heat flux density arising from the condensation of water vapor at the earth's surface
R	Heat flux density of effective radiation from the earth's surface, ie the Stefan-Boltzmann radiation minus the incoming radiation, $R = \sigma T^4 - R'$
R_0	Effective radiation with a clear sky
R_{10}	Effective radiation with an overcast sky
R_i	Effective radiation at sunset
R'	Incoming radiation from the atmosphere
R'_0	Incoming radiation from a clear sky
R'_{10}	Incoming radiation from an overcast sky
T	Surface temperature $^\circ A$
$\bar{T}$	Mean temperature $^\circ A$ for night
ΔT	Temperature change during the night
ΔT_g	Temperature change from Jacobs' Diagram
c_1	Specific heat of the soil in gram-calories
c_a	Specific heat of the air in gram-calories
d	Parameter that depends on the height of clouds
e	Vapor pressure in milibars of mercury
f	Rate of change of the effective radiation
k	Specific molecular conductivity of heat in soil cm/sec
k_a	Specific eddy conductivity of heat in air cm/sec
m	Fraction of cloud cover in tenths
t	Time in hours
ρ_1	Density of the soil gm-cm
ρ_a	Density of the air gm-cm
σ	Stefan-Boltzmann constant $= 0.817 \times 10^{-10}$ cal cm^{-2} min^{-1} deg^{-4}

I INTRODUCTION

1. Previous History.

A study of the problem of forecasting minimum temperature was started by the United States Weather Bureau at the turn of the century in order to assist the fruit and truck garden farmers in protecting their crops against frost. Special temperature forecasting stations were established and as a result numerous empirical mathematical formulas [12] were developed by these stations for their particular region. However the formulas developed were generally identical in principal, the minimum temperature usually being expressed as some function of the preceding temperature and hygrometric data.

During the last twenty years theoretical investigations of the nocturnal radiational fall of temperature at the earth's surface have been presented by several meteorologists. The theories of Brunt [3], Philipps [10], and Groen [7] are of especial interest and will be presented in Chapter II.

The development of Elsasser's Radiation chart [6] was a great step toward finding more accurately the temperature change due to nocturnal radiation. However to be of practical use more frequent soundings must be made.

2. Probelm involved.

The change in temperature during the night at a given location on the surface of the earth is caused by one or more of the following processes: (a) terrestrial radiation, (b) advection of warmer or colder air, (c) vertical heat transport due to turbulence, (d) local surface effects such as those of lakes, bays, islands and peninsulas, (e) evaporization cooling and condensation heating.

This paper deals only with the temperature changes caused by terrestrial radiation and vertical heat transport. For a discussion of the changes due to advection, local surface effects and evaporation cooling, the reader is referred to Byers [4].

3. Summary.

It is the purpose of this paper to develop a convenient method of forecasting minimum temperature using the theoretical and empirical knowledge as presented in the following chapter. It is to be noted that this work is based on data from the Patuxent River Naval Air Station on the Chesapeake Bay. Another purpose is to determine which of the theoretical and empirical rules might lead to a good forecast of nocturnal radiational cooling at a station having largely a maritime exposure.

1. Theoretical formulas.

The theories on nocturnal radiational fall of temperature as derived by Brunt [3], Philipps [10], and Groen [7], originate from the equation of the energy balance at the earth's surface.

$$B = R - A - C \qquad (1)$$

Where B is the heat flux density in the ground, R is the flux density of effective radiation from the earth's surface, A is the flux density due to eddy conduction in the air, and C is the heat flux denisty arising from the condensation of water vapor at the earth's surface; heat flux density is understood to mean the energy transport per unit area.

The heat flux density in the ground may be written:

$$B = -\rho_1 c_1 k_1 \left(\frac{\partial T}{\partial z}\right)_0 \qquad (2)$$

Where ρ_1 is the density, c_1, is the specific heat and k_1 is the specific molecular conductivity of heat of the soil.

In all the theories given below the following basic assumptions were made:

(1) The night is uniform in regard to air mass properties and cloudiness.

(2) The ground is homogeneous so that surface temperature variations are governed by the simple equation of conductivity of heat.

$$\frac{\partial T}{\partial t} = k_1 \frac{\partial^2 T}{\partial z^2} \qquad (3)$$

(3) The initial temperature of the soil is constant with depth.

$$T = \text{constant} \qquad (4)$$

(4) No condensation or evaporation occurs. ($C = 0$)

a. Brunt's formula.

In addition to the basic assumption given above, Brunt [3] made the further assumptions that R is constant with respect to time and that $A = 0$. From these given conditions Brunt obtained his well known formula:

$$\Delta T = \frac{2R\sqrt{t}}{\sqrt{\pi}\,\rho_1 c_1 \sqrt{K_1}} \tag{5}$$

b. Philipps' formula.

Philipps [10] also made the assumption that R is constant with respect to time; but he took into account the eddy conductivity term A, by assuming that there is a heat transport in the air obeying an equation similar to (4). With these given conditions he obtained the equation:

$$\Delta T = \frac{2R\sqrt{t}}{\sqrt{\pi}\,(\rho_1 c_1 \sqrt{K_1} + \rho_a c_a \sqrt{K_a})} \tag{6}$$

Where ρ_a is the density, c_a is the specific heat and k_a is the specific eddy conductivity of heat in the air.

c. Groen's formula.

Groen [7] in developing his formula, assumed a variable R, but otherwise made the same assumptions as Brunt. With these assumptions he arrived at the following formula:

$$\Delta T = \frac{R_i}{f}\left[\frac{2}{\sqrt{\pi}}\left(f\frac{\sqrt{t}}{\rho_1 c_1 \sqrt{K_1}}\right) - \left(f\frac{\sqrt{t}}{\rho_1 c_1 \sqrt{K_1}}\right)^2 + \frac{4}{3\sqrt{\pi}}\left(f\frac{\sqrt{t}}{\rho_1 c_1 \sqrt{K_1}}\right)^3 \cdots\cdots\right] \tag{7}$$

Where R_i is the initial value of the effective radiation at sunset and f is the rate of change of the effective radiation. If the rate of change of the radiation (f) is small so that second and higher order terms may be neglected, equation (7) reduces to Brunt's formula.

d. Formula for variable R with α not equal to zero.

It is easy to see that if we make the same assumption as for Philipps' formula, but use a variable R as Groen suggested we arrive at the following formula:

$$\Delta T = \frac{R_i}{f}\left[\frac{2}{\sqrt{\pi}}\left(\frac{f\sqrt{t}}{\rho_1 c_1\sqrt{K_1}+\rho_a c_a\sqrt{K_a}}\right) - \left(\frac{f\sqrt{t}}{\rho_1 c_1\sqrt{K_1}+\rho_a c_a\sqrt{K_a}}\right) \ldots\ldots\right] \qquad (8)$$

In this case if the rate of change of the radiation is small, the equation reduces to Philipps' formula.

2. Empirical values of the soil and air constants.

The so-called soil constants (ρ_1 , c_1 , k_1 ,) in the above formula are unfortunately only constant for a given soil under given conditions. Therefore these constants vary with change of soil from one region to another; and even from day to day when there is a change of moisture content of the ground. Values of these soil constants have been computed for different soils under different conditions and are given in tables 1 to 5 of Jacobs paper [9] . However the soils surrounding a given region consist of many different types and they all influence the air temperature that is measured at the thermoscreen.

In using Philipps' formula it is also necessary to know the air constants (ρ_a , c_a , K_a).

The values of density (ρ_a) and specific heat, (c_a) at constant pressure, are easily found from the given conditions of the air. The eddy conductivity (k_a) however, varies with the wind velocities and various empirical values have been obtained for different wind velocities. Taylor's estimates [11] of the magnitude of k_a as obtained over the Grand Banks of Newfoundland and Dobson's estimates [5] of k_a over land are given in Table I.

Table I. Values of k_a for various winds.					
Wind (Beauford Scale)	1	2	3	5	7
Taylor's value cm^2/sec	1×10^3	2×10^3	3×10^3		
Dobson's value cm^2/sec			2.72×10^4	4.51×10^4	5.38×10^4

It may be noted that Taylor's values of k_a are very much smaller than those of Dobson's. The difference may be ascribed to stability in the surface inversion. Thus it is evident that the value of k_a will vary with different regions and different seasons.

Philipps using values of ρ_a and c_a for dry air at standard pressure and temperature computed values of the combined soil and air constants for different soils, which are given in Table II.

3. Empirical values of the effective radiation on clear nights.

The effective radiation R is understood to be the outgoing radiation of the earth minus the incoming radiation from the atmosphere R'.

$$R = \sigma T^4 - R' \qquad (9)$$

There have been several empirical formulas derived, based on surface conditions of vapor pressure (e), and temperature. The best known formulas are those of Angström [1], Brunt [3], and Elsasser [6], which are given below with their constants.

Angstrom's $$R' = \sigma T^4 \left(a - b \cdot 10^{-\gamma e} \right) \qquad (10)$$

$a = 0.25 \qquad b = 0.32 \qquad \gamma = 0.052$

Brunt's $$R' = \sigma T^4 \left(a + b\sqrt{e} \right) \qquad (11)$$

$a = 0.44 \qquad b = 0.080$

Elsasser's $$R' = \sigma T^4 \left(a + b \log e \right) \qquad (12)$$

$a = 0.21 \qquad b = 0.22$

In spite of apparent differences these formulas give about the same results for observed values of vapor pressure and temperature. As Brunt's

TABLE II. VALUES OF $(c_1\rho_1\sqrt{k_1} + c_a\rho_a\sqrt{k_a})10^2$ FOR DIFFERENT EDDY CONDUCTIVITIES AND FOR DIFFERENT SOIL TYPES. $c_a = 0.2396$ gram-calories $\rho_a = 1.3 \cdot 10^{-3}$ gram-cm^{-3} Eddy conductivity (cm^2/sec)

Soil type	0	7.7	38.5	77	308	538	1000	2000	3000	$2.72 \cdot 10^4$	$4.51 \cdot 10^4$
Sand	3.91	4.00	4.11	4.19	4.46	4.64	4.90	5.31	5.62	9.05	10.54
Moist sand	3.28	3.37	3.48	3.56	3.83	4.01	4.27	4.68	4.99	8.42	9.91
Sand stone	6.99	7.08	7.19	7.27	7.54	7.72	7.98	8.39	8.70	12.13	13.62
Rock gravel	4.72	4.81	4.92	5.00	5.27	5.45	5.71	6.12	6.43	9.86	11.35
Granite	7.04	7.13	7.24	7.32	7.59	7.77	8.03	8.44	8.75	12.18	13.67
Loamy soil	4.90	4.99	5.10	5.18	5.45	5.53	5.89	6.30	6.61	10.04	11.35
Moor	4.23	4.32	4.43	4.51	4.78	4.96	5.22	5.63	5.94	9.37	10.86

formula will be used in the analysis, the values of the constants a and b as determined by various investigators, are given in Table III.

Table III. Values for a and b for Brunt's Radiation Formula determined by various investigators.

	a	b	$\frac{1-a}{b}$
Dines (Benson)	0.52	0.065	7.32
Asklof (Upsala)	0.43	0.082	6.95
Angstrom (Bassour)	0.48	0.058	8.97
Boutarie (France)	0.60	0.042	9.53
Robitsch (Lindenberg)	0.34	0.110	6.00
Ramanathan and Desai (Poona)	0.47	0.061	8.69
Means	0.44	0.080	7.0

The best method for determing the incoming radiation from the atmosphere is the radiation chart method, since it is based on the upper air moisture content as obtained by soundings. Elsasser's [6] Radiation chart is admirably suited to finding this value of R' but it is necessary to have a sounding near sunset to be of use in forecasting. The effective radiation, R , can be obtained from the chart just as conveniently as the radiation from the atmosphere. If Groen's formula is to be used it is also necessary to take at least one more sounding during the night in order to find the rate of change of the effective radiation. As to the method of construction and use of the chart the reader is referred to papers by Elsasser [6].

4. Nocturnal radiation with cloudy skies.

With an overcast sky the incoming radiation from the atmosphere is greatly increased because the cloud layer absorbs and radiates as a black body at the temperature of the cloud base. Therefore the effective radiation from the earth's surface is greatly decreased with a resulting small temperature change during the night. As the temperature normally decreases with increasing height, the effective radiation will increase with the height of the cloud.

The radiation from a partly cloudy sky may in general be written as follows:

$$R' = R_0' + m\ (R_{10}' - R_0') \qquad (13)$$

where R_0' is the radiation from a clear sky, R_{10}' is the radiation from an overcast sky, and m is the fractional amount of cloudiness measured in tenths of sky covered by cloud. From equation (9) the above equation (13) may be transformed into terms of the effective radiation to give the following equation:

$$R = R_0 + m\ (R_{10} - R_0) \qquad (14)$$

Where R_0 is the effective radiation with a clear sky and R_{10} with an overcast sky.

By using a radiation chart it is very easy to find the effective radiation from both a clear and overcast sky as we know the temperature at the cloud height from the sounding.

For an empirical formula for effective radiation with a cloudy sky, using surface data, we assume that:

$$R_{10} = (1 - d)\ R_0 \qquad (15)$$

where d varies with the height of clouds.

Philipps [10] computed theoretical values for d assuming a surface temperature of 294°A and a lapse rate of 5°/km and obtained the following results:

	0km	2km	5km	8km
d	1	0.83	0.62	0.45

The mean of Asklöf's [2] and Angström's [1] observational data gave the following values:

	1.5km	3km	7km
d	0.86	0.75	0.20

These observed values agree very closely with the calculated values of Philipps except for high clouds. The reason for this discrepancy is that the clouds at this height are comparatively thin and do not radiate as a black body. Therefore above 5km it would be better to use observed values of d in finding the effective radiation.

From equation (14) and (15) we arrive at the following empirical formulas for partly cloudy skies:

$$R = R_0 (1 - md) \tag{16}$$

III. A STATISTICAL ANALYSIS OF THE NOCTURNAL COOLING

As the rate of change of the effective radiation during the period from sunset to sunrise is normally small, except with very calm conditions, we will use the average effective radiation, R, and assume it to be constant throughout this period.

From the theoretical and empirical knowledge presented in the previous chapter it is evident that the formula for the change in temperature should be of the following form:

$$\Delta T = \frac{2R\sqrt{t}}{\sqrt{\pi}(\rho_1 c_1 \sqrt{K_1} + \rho_a c_a \sqrt{K_a})} = F R \sqrt{t} \tag{17}$$

Where $F = \dfrac{2}{\sqrt{\pi}(\rho_1 c_1 \sqrt{K_1} + \rho_a c_a \sqrt{K_a})}$

Thus the constant F will vary with different soils, condition of the soil and different wind velocities.

In solving for the effective radiation we may use either a radiation chart, if soundings are available, or one of the empirical formulas for the effective radiation if only surface data is available. We therefore have two distinct methods of solving for the temperature change.

1. Radiation formula method.

In using the radiation formula method we may use any one of the radiation formulas (10, 11, 12) presented in the preceding chapter. For this analysis, Brunt's formula (11) was used for its convenience in computation. For clear skies the formula for change in temperature will then be as follows:

$$\Delta T = F \sigma \bar{T}^4 \left(1 - a - b\sqrt{e} \right) \sqrt{t} \tag{18}$$

Let $D = 280^4 \sigma F$

$$\Delta T = D \left(\frac{\bar{T}}{280} \right)^4 \left(1 - a - b\sqrt{e} \right) \sqrt{t} \tag{19}$$

Let $G = D(1 - a)$ $\qquad H = Db$

$$T = \left(\frac{\bar{T}}{280}\right)^4 (G - H\sqrt{e})\sqrt{t} \qquad (20)$$

Using observed values of ΔT, $\bar{T}$, e and t we can solve for G and H for given wind velocities. Using the method of least squares, values of G and H were computed for the months of August to November over a period of three years. From the values obtained, it was found convenient to group August with September, and October with November. The results obtained for these periods at different average wind velocities are given in Table IV.

Table IV. Computed values of G and H for various wind velocities.

Wind speed (Beaufort Scale)	Number of Observations	G	H	G/H
	August and September			
2	16	7.533	1.010	7.45
3	11	3.163	1.244	2.54
	October and November			
2	18	5.627	.710	7.92
3	12	6.626	1.284	5.16
4	5	10.251	2.92	3.51

Comparing the ratio G/H with the ration $\frac{1 - a}{b}$ from Brunt's formula, (Table III), it is found that they agree quite closely for low wind velocities. The deviations with higher wind velocities are attributed to the smaller number of observations and the greater possibility of advection at these higher wind velocities.

The mean values of a and b will therefore be used with a parameter D that varies with wind velocity and soil conditions. Thus our equation becomes:

$$\Delta T = D \left(\frac{\bar{T}}{280}\right)^4 (0.56 - 0.080\sqrt{e}) \sqrt{t} \qquad (21)$$

Jacobs [8] in his paper has computed a table and constructed a graph based on a mean temperature of 280° A and a D of 12.30 $^{\circ}C/hr^{\frac{1}{2}}$, which gives the temperature fall at a given time after sunset. He has also computed a table and graph which gives a correction factor for mean temperatures $\bar{T}$ differing from 280°A.

Using Jacobs' diagram (Figure 1), we will obtain a ΔT_g based on a fixed D of 12.30 $^{\circ}C/hr^{\frac{1}{2}}$. In order to find a correction factor for different soil and wind conditions we will solve the following equation for M.

$$\Delta T = M\Delta T_g \quad \text{where } M = \frac{D}{12.3} \qquad (22)$$

Figure 2 is a graphical plot of ΔT_g versus ΔT for the months of August and September with wind velocity of Beaufort force 2.

From observed data of ΔT, T_d, $\bar{T}$ and t we obtain values of M as given in Table V.

Table V. Computed values of M.

Wind speed (Beaufort scale)	Number of Observations	M	Standard Deviation
August and September			
2	16	.655	.0866
3	11	.552	.0700
October and November			
2	18	.486	.0980
3	12	.350	.0837
4	5	.287	.1072

The decrease in M from August to November indicates a greater percentage of moisture in the soil in the later months. The decrease in M with increasing wind velocity indicates an eddy conductivity in the air of the order given by Dobson's data in Table I.

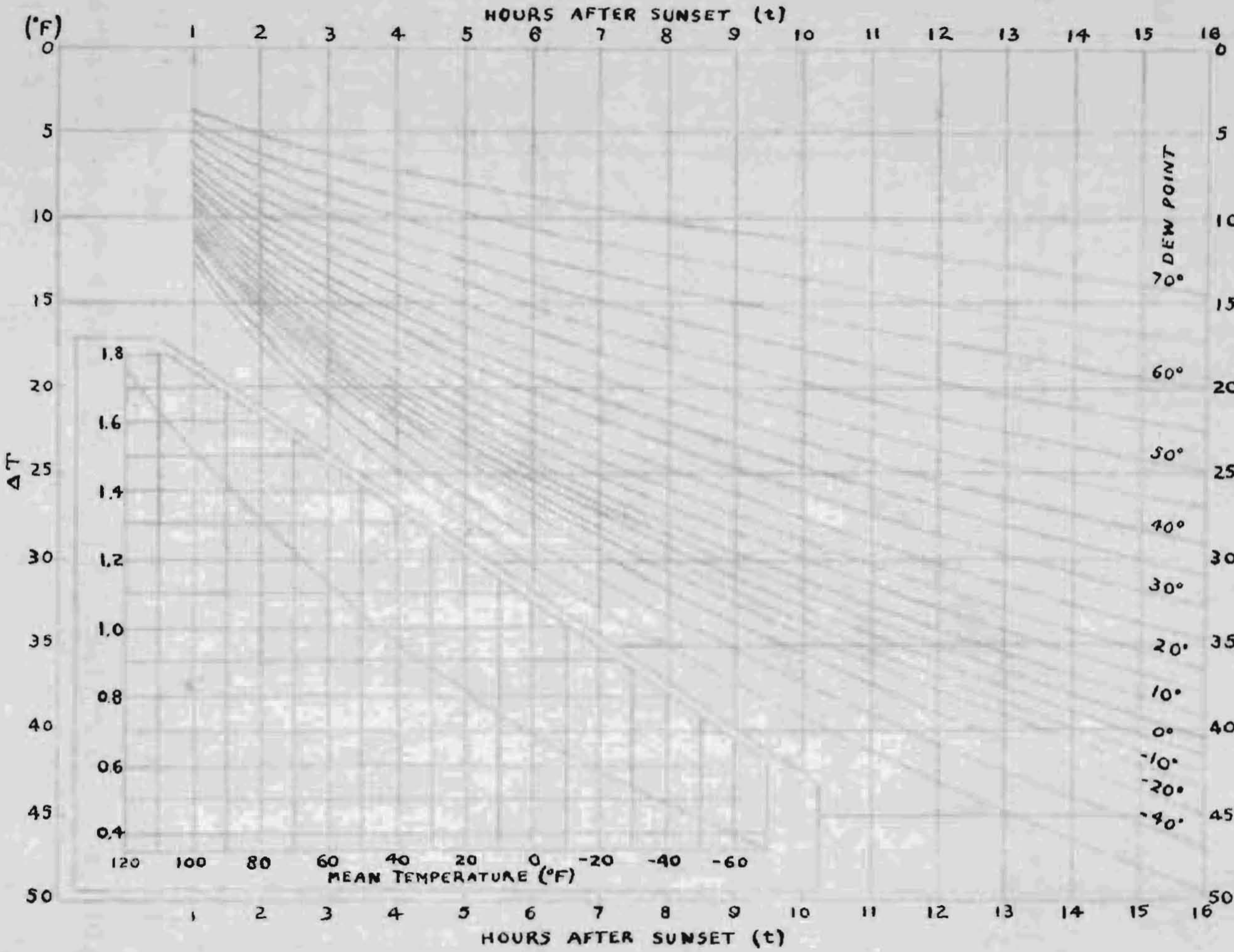


FIGURE 1. JACOBS' DIAGRAM

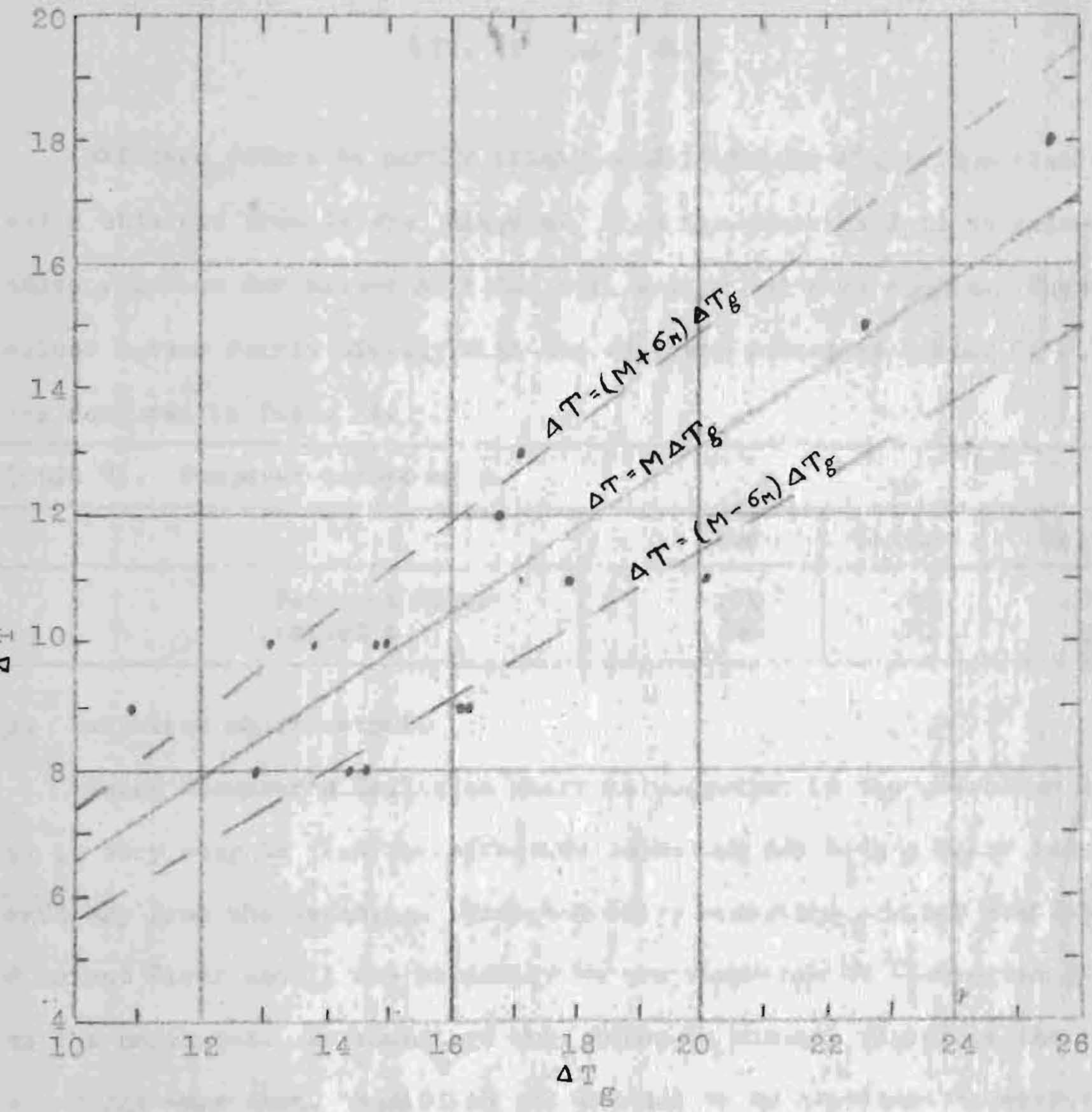


FIGURE 2. GRAPH OF ΔT_g VERSUS ΔT FOR THE MONTHS OF AUGUST AND SEPTEMBER WITH A WIND VELOCITY OF BEAUFORT FORCE 2.

With the values of M given in Table V we can solve for the effect of cloud cover by the use of the empirical equations (16), (17), and (22) given in the previous chapter. The result is:

$$\Delta T = (1 - md) \ M\Delta T_g \tag{23}$$

ΔT here refers to partly cloudy conditions and ΔT_g is the clear sky value obtained from Jacobs' diagram. From the observed data we solved the above equation for values of d for low, medium and high clouds. These values agreed fairly closely with the observed values of Asklöf [2], and are compared in Table VI.

Table VI. Computed values of d.

	Low	Middle	High
Patuxent River	.92	.78	.09
Asklöf's	.86	.75	.20

2. Radiation chart method.

Using Elsasser's Radiation chart as suggested in the preceding chapter, it is very easy to find the effective radiation for both a clear and overcast sky from the sounding. Unfortunately, soundings are not available for Patuxent River and it was necessary to use soundings at Washington 50 miles to the northwest. No soundings were taken at sunset, therefore the 2300 EST soundings were used. Radiation was assumed to be constant throughout the night.

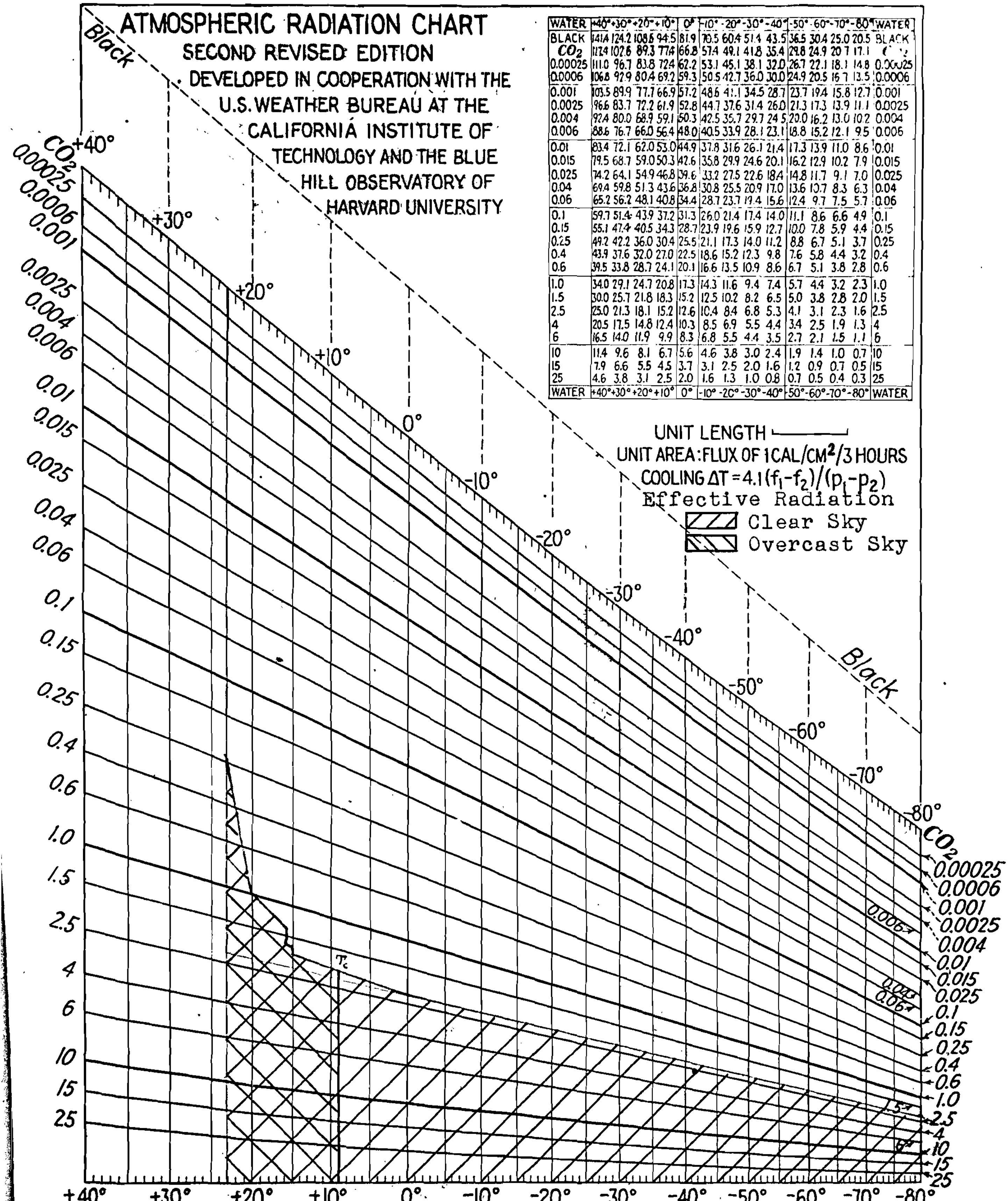
In solving for the effective radiation we will use formula (14) for low and middle clouds.

$$R = R_0 + m(R_{10} - R_0) \tag{14}$$

However in view of the fact that high clouds are relatively thin and do not radiate as a black body, we will use the empirical formula (16) for

TABLE VII. TYPICAL COMPUTATIONS OF u FROM GIVEN SOUNDING							
1 p(mb)	2 T^oC	3 w	4 Mean w	5 Δp	6 $\sqrt{p/p_o}$	7 moist.	8 u
1018	23	13.5					0.00
			13.1	18	1.00	.24	
1000	23	12.7					0.24
			11.6	64	.98	.73	
936	20	10.4					0.97
			9.8	46	.96	.43	
890	16	9.3					1.40
			7.8	26	.94	.19	
864	16	6.2					1.59
			7.1	14	.93	.09	
850	15	7.9					1.68
			6.2	20	.92	.11	
830	15	4.6					1.79
			4.4	65	.89	.25	
765	9	4.2					2.04
			3.5	21	.87	.06	
744	8	2.8					2.10
			2.4	16	.86	.03	
728	9	1.9					2.13
			1.0	28	.84	.02	
700	8	-					2.15
			0.9	65	.82	.05	
635	5	1.7					2.20
			1.6	60	.77	.07	
575	0	1.4					2.27

FIGURE 3. TYPICAL (u,T) CURVE PLOTTED ON AN ELSASSER RADIATION CHART.



WATER	+40°	+30°	+20°	+10°	0°	-10°	-20°	-30°	-40°	-50°	-60°	-70°	-80°	WATER
BLACK	141.4	124.2	108.6	94.5	81.9	70.5	60.4	51.4	43.5	36.5	30.4	25.0	20.5	BLACK
CO_2	112.4	102.6	89.3	77.4	66.8	57.4	49.1	41.8	35.4	29.8	24.9	20.7	17.1	CO_2
0.00025	111.0	96.7	83.8	72.4	62.2	53.1	45.1	38.1	32.0	26.7	22.1	18.1	14.8	0.00025
0.0006	106.8	92.9	80.4	69.2	59.3	50.5	42.7	36.0	30.0	24.9	20.5	16.7	13.5	0.0006
0.001	103.5	89.9	77.7	66.9	57.2	48.6	41.1	34.5	28.7	23.7	19.4	15.8	12.7	0.001
0.0025	96.6	83.7	72.2	61.9	52.8	44.7	37.6	31.4	26.0	21.3	17.3	13.9	11.1	0.0025
0.004	92.4	80.0	68.9	59.1	50.3	42.5	35.7	29.7	24.5	20.0	16.2	13.0	10.2	0.004
0.006	88.6	76.7	66.0	56.4	48.0	40.5	33.9	28.1	23.1	18.8	15.2	12.1	9.5	0.006
0.01	83.4	72.1	62.0	53.0	44.9	37.8	31.6	26.1	21.4	17.3	13.9	11.0	8.6	0.01
0.015	79.5	68.7	59.0	50.3	42.6	35.8	29.9	24.6	20.1	16.2	12.9	10.2	7.9	0.015
0.025	74.2	64.1	54.9	46.8	39.6	33.2	27.5	22.6	18.4	14.8	11.7	9.1	7.0	0.025
0.04	69.4	59.8	51.3	43.6	36.8	30.8	25.5	20.9	17.0	13.6	10.7	8.3	6.3	0.04
0.06	65.2	56.2	48.1	40.8	34.4	28.7	23.7	19.4	15.6	12.4	9.7	7.5	5.7	0.06
0.1	59.7	51.4	43.9	37.2	31.3	26.0	21.4	17.4	14.0	11.1	8.6	6.6	4.9	0.1
0.15	55.1	47.4	40.5	34.3	28.7	23.9	19.6	15.9	12.7	10.0	7.8	5.9	4.4	0.15
0.25	49.2	42.2	36.0	30.4	25.5	21.1	17.3	14.0	11.2	8.8	6.7	5.1	3.7	0.25
0.4	43.9	37.6	32.0	27.0	22.5	18.6	15.2	12.3	9.8	7.6	5.8	4.4	3.2	0.4
0.6	39.5	33.8	28.7	24.1	20.1	16.6	13.5	10.9	8.6	6.7	5.1	3.8	2.8	0.6
1.0	34.0	29.1	24.7	20.8	17.3	14.3	11.6	9.4	7.4	5.7	4.4	3.2	2.3	1.0
1.5	30.0	25.7	21.8	18.3	15.2	12.5	10.2	8.2	6.5	5.0	3.8	2.8	2.0	1.5
2.5	25.0	21.3	18.1	15.2	12.6	10.4	8.4	6.8	5.3	4.1	3.1	2.3	1.6	2.5
4	20.5	17.5	14.8	12.4	10.3	8.5	6.9	5.5	4.4	3.4	2.5	1.9	1.3	4
6	16.5	14.0	11.9	9.9	8.3	6.8	5.5	4.4	3.5	2.7	2.1	1.5	1.1	6
10	11.4	9.6	8.1	6.7	5.6	4.6	3.8	3.0	2.4	1.9	1.4	1.0	0.7	10
15	7.9	6.6	5.5	4.5	3.7	3.1	2.5	2.0	1.6	1.2	0.9	0.7	0.5	15
25	4.6	3.8	3.1	2.5	2.0	1.6	1.3	1.0	0.8	0.7	0.5	0.4	0.3	25
WATER	+40°	+30°	+20°	+10°	0°	-10°	-20°	-30°	-40°	-50°	-60°	-70°	-80°	WATER

high clouds.

$$R = (1 - md)R_o \text{ where } d = 0.20 \qquad (16)$$

The use of the radiation chart is explained in detail in Elsasser's paper [6], but in order to show how the effective radiation was obtained, Table VII and Figure 3 are given as a typical example. The first three columns are taken directly from the sounding. The next three were obtained from this data. The effective amount of moisture in each layer, column seven, is the product of the figures of columns 4, 5, and 6 divided by 1000. The moisture thickness, column 8, is obtained by adding the values in column seven from the surface upward. The (u, T) curve of state of the water vapor atmosphere is then plotted on the radiation chart. The effective radiation is then obtained from the chart by use of table on chart and measuring irregular areas as indicated in Figure 3. With an overcast sky, the cloud is assumed to radiate as a black body at the temperature at the base of the cloud and the effective radiation would be as indicated in Figure 3.

Knowing ΔT and t from observations and R from the radiation chart, we can solve equation (17) for the constant F for different wind velocities and soil conditions.

Computing F for the month of August we obtained values of F as given in Table VIII.

Table VIII. Computed values of F.

Wind Velocity (Beaufort scale)	F	σ_F
2	.154	.0224
3	.146	.0387

3. Results.

From this analysis we find that both methods give results with about the same degree of accuracy. The radiation formula method however, is much simpler and faster than the radiation chart method. Surface data only is needed in the radiation formula method, while the radiation chart method requires a sounding. Computation with the formula method is greatly reduced by the use of graphs, while the chart method involves measurement of areas as well as long computation. Another disadvantage of the chart method is the scarcity of radiosonde data and of low level soundings, both in regard to location and to time.

Soundings could be used in conjuction with the radiation formula method to find the temperature of the cloud base, and thus a more accurate value of the constant d.

From equation (15) we have:

$$d = 1 - \frac{R_{10}}{R_o} \tag{24}$$

The soundings analyzed were all similar to the typical example given in Figure 3 in that the maximum moisture thickness was about 2.5 grams per square centimeter and that the area between the curve of state (u,T) and the maximum moisture thickness was approximately one calorie per square centimeter per three hours. Table IX gives values of d for different mean surface temperature and cloud temperatures, which were computed assuming conditions as given above.

Table IX. Values of d.

Cloud Temperature	Mean surface temperature						
	40	30	20	10	0	-10	-20
40	1.0						
30	.819	1.0					
20	.696	.812	1.0				
10	.584	.682	.796	1.0			
0	.484	.565	.660	.778	1.0		
-10	.400	.466	.544	.642	.765	1.0	
-20	.322	.376	.440	.519	.617	.737	1.0
-30	.261	.304	.356	.421	.500	.597	.723
-40	.203	.237	.277	.327	.390	.464	.563

IV. FORECASTING THE MINIMUM TEMPERATURE.

From the results of the analysis in the preceeding chapter, it is evident that the most convenient method of forecasting the minimum temperature is by the radiation formula method.

1. Procedure.

We enter Jacobs' diagram (Figure 1) with the dew point and the time between sunset and sunrise and find ΔT_g . This term ΔT_g is then corrected for the soil and wind conditions by multiplying by the value M as previously computed from past data.

To correct for expected cloud cover we multiply the product obtained above by the correction factor (1-md) where m is the amount of cloud cover and d is a constant depending upon the height of the clouds. Table VI will be used when soundings are not available, and Table VIII will be used when soundings are available, to obtain the value of d.

Referring again to Jacobs' diagram (Figure 1) we find a correction factor for the mean temperature during the night, which when multiplied by the product obtained in the previous step will give the change in temperature according to the radiation formula.

This value of ΔT is then subtracted from its observed surface temperature at sunset to obtain the minimum temperature during the night.

2. Remarks.

In computing values of M for a station, it is recommended that a day to day record be kept of the value of M that would have given a correct minimum temperature, and to use the mean D for the expected wind velocity, as obtained from the ten previous days having that velocity, for forecasting purposes. With this method it is believed that a more representative value of D will be obtained. By keeping a daily record it will also be possible to separate very moist conditions of the ground from the average condition during each month.

When advection of warm or cold air is taking place this method will not be valid unless some method, such as Byers' [4] is used to obtain the temperature change due to advection. With advection of moist or dry air an estimate must be made as to the expected average value of the dew point.

Care must also be exercised to eliminate changes in temperature due to local effects. In this analysis the station was on a peninsula and had water to the north, east, and south. However for the period investigated wind direction did not appear to influence the temperature drop during the night.

When the minimum temperature as computed by the above method is equal to or less than the dew point at sunset, it is a good indication that fog might form. In this case the minimum temperature would be near the dew point.

3. Results.

Following the procedure indicated above, minimum temperatures were computed for the period from August to November 1946 at Patuxent River, Maryland. Figure 4 is a graph showing the deviation of the computed from the observed values of the minimum temperature for the period. Table X is given to show more clearly the deviations by comparing the computed values of ΔT with the observed values of ΔT. This graph and table indicate that 75% of the forecasted minimum temperature are within 2° of the observed minimum temperature and that 92% are within 3°. A closer inspection of the changes in temperature due to advection, local effect, or condensation should increase this accuracy.

TABLE X. COMPARISON OF FORECASTED ΔT WITH OBSERVED ΔT.

		ΔT OBSERVED									
		5	6	7	8	9	10	11	12	13	14
ΔT FORECASTED	5	2									
	6		1		1						
	7			2							1
	8	3	2	5	2	1	2				1
	9		2	1	1		1	1	1		
	10				2	3	1		3	1	
	11				4	3	2	1			
	12			1		1	2		1		
	13							1	2		1
	14			1			1	1			1

4. Conclusions.

Although the above method gives a good approximate value for the temperature change from sunset to sunrise, there still exists the problem of finding the temperature change from forecast time to sunset. A similar method of finding the temperature change using the radiation received from the sun has been suggested but the variations of the solar radiation received at the earth's surface makes this method too complicated to use. If the time of making the forecast is after the maximum temperature it should be possible to make a good estimate of the temperature at sunset from the temperature trace. Thus the radiation formula method of finding the minimum temperature is of practical use only when forecasting at or after time of maximum temperature.

This method of finding the change in temperature may also be used to predict the time of formation of a radiational fog, by finding the time

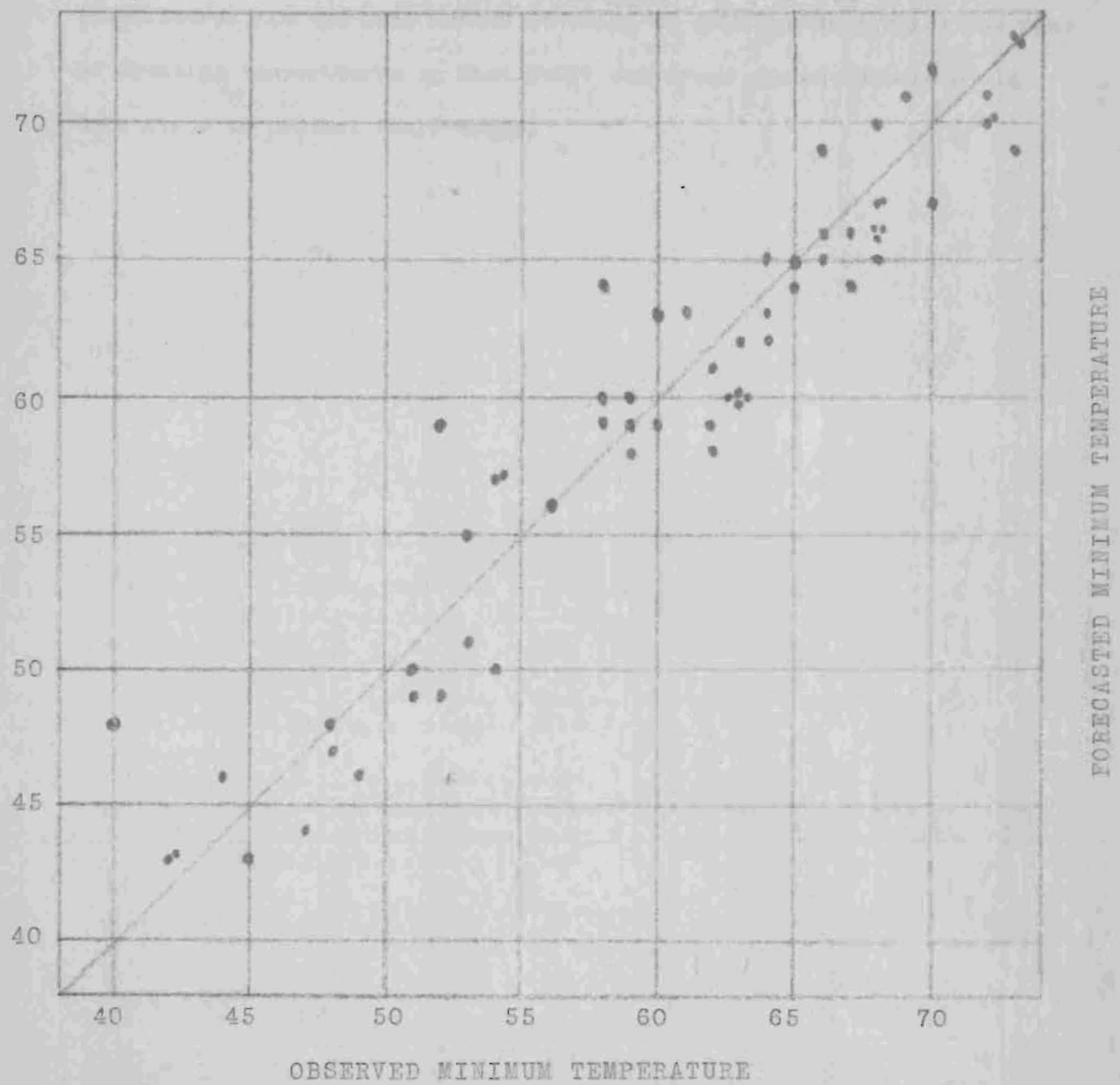


FIGURE 4. GRAPH OF FORECASTED VERSUS OBSERVED MINIMUM TEMPERATURES

when the surface temperature will be equal to the expected dew point temperature. In the same manner it would be possible to predict the time of freezing temperatures so that fruit and truck garden farmers could take steps to protect their crops.

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